MATHEMATICAL MODELING OF THE PROCESS OF HEAT AND MOIST TREATMENT OF CONCRETE AND REINFORCED-CONCRETE PRODUCTS

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A system of equations describing the dynamics of the process of heat and moist treatment of concrete and reinforced-concrete products in pit-type, steam-curing chambers is given. As a result of a computational experiment, the influence of the parameters of a technological regime on the basic performance figures of the process is studied.

Heat and moist treatment (steam curing) in special thermal plants, i.e., pit-type, steam-curing chambers of periodic action (Fig. 1), enjoys wide application in industry to accelerate the hardening of concrete and of reinforced-concrete products.

The basic elements of a pit-type chamber are walls 1, floor 2, the removable cover 3, and the systems of ventilation and steam lines equipped with control and stop valves.

The walls and the floor of the chamber are usually made of reinforced concrete with a thickness of 250 to 400 mm. Such walls are strong, not very heat-conducting, and quite impenetrable to a steam-air mixture. However, the high consumption of heat on heating them and high thermal inertia are the drawbacks.

In pit-type chambers, the pressure difference of the steam-air mixture and the atmosphere does not exceed 8–10 mm Hg. To produce a low excess (negative) pressure one installs in the chamber a so-called return pipe, which connects the internal volume of the chamber with the atmosphere. On the return pipe, there is a hydraulic valve which enables one to maintain the prescribed pressure in the chamber.

The interior surfaces of the chamber and concrete products are both heated and cooled in the course of heat and moist treatment; the heat exchange is complicated by mass-exchange processes (condensation and evaporation of moisture from the open surface of the products).

The development of a mathematical model of the process of heat treatment and of efficient algorithms of controlling this process is a pressing problem at present whose solution contributes to a reduction in the consumption of expensive raw materials, expenditure of energy, and the specific quantity of metal in finished products and to an increase in the output of products and hence a reduction in manufacturing cost and an increase in the profit of enterprises. There are mathematical models of heat exchange in the case of heat treatment of concrete, for example [1–4], but they give no way of describing systematically the heat and moist treatment of reinforced-concrete products and only reflect its individual processes.

Thus, the problem of construction of a mathematical model suitable for the purposes of simulation investigations and optimization of heat and moist treatment has not yet been solved.

The proposed mathematical model of the process of heat and moist treatment of reinforced-concrete products embodies the modular principle and consists of interrelated mathematical descriptions of the processes occurring in the products, the steam-air volume of the plant, and the enclosing structures (cover, floor, walls) of the chamber.

The mathematical model of the processes occurring in reinforced-concrete slabs involves the equation to determine the ultimate strength of concrete of prescribed composition

$$\frac{d\zeta_{\text{concr}}^{\text{pr}}}{d\tau} = a_1 \exp\left(\frac{-a_2}{\overline{T}_{\text{concr}}^{\text{pr}}}\right)\left(\overline{\zeta_{\text{concr}}^{\text{pr}}(\nu)} + \zeta_{\text{concr}0}^{\text{pr}} - \zeta_{\text{concr}}^{\text{pr}}\right), \quad \tau > 0\,; \quad \zeta_{\text{concr}}^{\text{pr}} = \zeta_{\text{concr}0}^{\text{pr}}\,, \quad \tau = 0\,, \tag{1}$$

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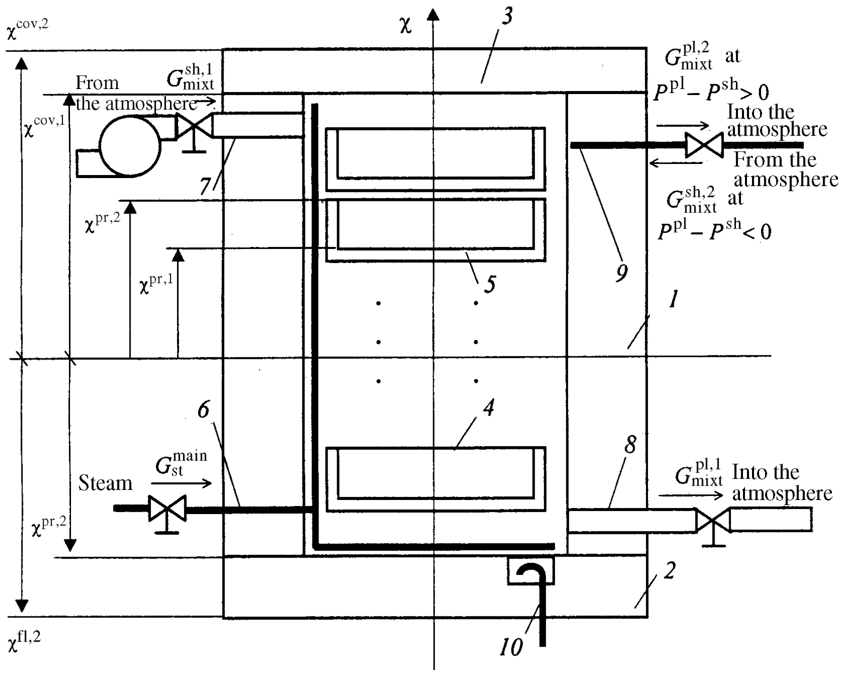

Fig. 1. Flow diagram of the process of heat and moist treatment of reinforced-concrete products: 1) walls; 2) floor; 3) cover; 4) products; 5) forms; 6) steam line; 7) influent channel; 8) exhaust duct; 9) return pipe; 1) steam trap.

and the one-dimensional heat-conduction equation

$$\rho^{pr} c^{pr} \frac{\partial T^{pr}}{\partial \tau} = \frac{\partial}{\partial \chi}\left(\lambda^{pr} \frac{\partial T^{pr}}{\partial \chi}\right) + b m^{pr}_{cem0} \frac{d\nu}{d\tau}\,, \quad \chi^{pr,1} < \chi < \chi^{pr,2}\,, \quad \tau > 0\,,$$

$$T^{pr} = T^{pr}_0\,, \quad \chi^{pr,1} < \chi < \chi^{pr,2}\,, \quad \tau = 0\,; \quad \lambda^{pr} \frac{\partial T^{pr}}{\partial \chi} = q^{pr,i}\,, \quad \tau \ge 0\,, \quad \chi = \chi^{pr,i}\,, \quad i = 1, 2\,. \tag{2}$$

The heat fluxes $q^{pr,i}$, $i = 1, 2$, take into account different situations of heat and mass exchange [2] which occur in the process of heat and moist treatment of reinforced-concrete products:

$$q^{pr,1} = \begin{cases} q_1^{pr,1} + r_{st}^{pr,1} g_{st}^{pr,1}\,, & T^{pl} > T^{pr,1}\,, \quad P^{pl}_{st} > P^{pr,1}_{sat}\,, \\ q_1^{pr,1}\,, & T^{pl} > T^{pr,1}\,, \quad P^{pl}_{st} \le P^{pr,1}_{sat}\,, \\ -q_2^{pr,1}\,, & T^{pl} \le T^{pr,1}\,, \end{cases} \qquad q^{pr,2} = \begin{cases} q_1^{pr,2} + r_{st}^{pr,2} g_{st}^{pr,2}\,, & T^{pl} > T^{pr,2}\,, \quad P^{pl}_{st} > P^{pr,2}_{sat}\,, \\ q_1^{pr,2} - r_{st}^{pr,2} g_{water}^{pr,2}\,, & T^{pl} > T^{pr,2}\,, \quad P^{pl}_{st} \le P^{pr,2}_{sat}\,, \\ -q_2^{pr,2} - r_{st}^{pr,2} g_{water}^{pr,2}\,, & T^{pl} \le T^{pr,2}\,. \end{cases} \tag{3}$$

According to the Newton–Richmann law, the quantities $q_1^{pr,i}$ and $q_2^{pr,i}$, $i = 1, 2$, appearing in (3) are determined from the formulas

$$q_1^{pr,i} = \alpha_1^{pr,i} (T^{pl} - T^{pr,i})\,, \quad q_2^{pr,i} = \alpha_2^{pr,i} (T^{pr,i} - T^{pl})\,, \quad i = 1, 2\,. \tag{4}$$

According to the Dalton law, the flow rate of the condensate vapor $g_{st}^{pr,1}$ and $g_{st}^{pr,2}$ per unit of, respectively, the lower and upper surfaces of a product is found in the form

$$g_{st}^{pr,i} = \beta_{st}^{pr,i} (P^{pl}_{st} - P^{pr,i}_{sat})\,, \quad i = 1, 2\,. \tag{5}$$

The intensity of evaporation of moisture from the open surface of hardening concrete is lower than from the surface of free water, since the liquid phase in the concrete is presented by a complex mineral solution. According to [5], the specific flow rate of the water evaporating from the open surface of products is found as

$$g^{pr,2}_{water} = \begin{cases} 0\,, & \xi P^{pr,2}_{sat} \le P^{pl}_{st}\,, \\ \beta^{pr,2}_{water}\,(\xi P^{pr,2}_{sat} - P^{pl}_{st})\,, & \xi P^{pr,2}_{sat} > P^{pl}_{st}\,, \end{cases} \qquad \xi = 0.091 \cdot 10^{293/T^{pr,2}}\,.$$

The density of the products ρ^{pr} can be considered to be a constant dependent only on the composition of reinforced-concrete products. The coefficients of heat capacity c^{pr} and thermal conductivity λ^{pr} of reinforced-concrete products are determined by the following general dependences [5]:

$$c^{pr} = c^{pr}_{concr}\,(T^{pr}, \nu)\,, \quad \lambda^{pr} = \lambda^{pr}_{concr}\,(T^{pr}, \nu)\,. \tag{6}$$

To describe the process of hydration of cement (in particular, to determine ν) we use the basic conclusions of the mathematical model of [3]. As of today, this model takes into account most completely the basic technological factors, while the calculation results are in good agreement with experimental data.

A distinctive feature of steam-curing chambers is the high thermal inertia and accumulating power of the enclosing structures of a chamber. These properties are taken into account by the heat-conduction equations to describe the processes of heat transfer in the cover and in the floor of the chamber:

$$\frac{\partial T^i}{\partial \tau} = \lambda^i/(c^i\rho^i)\,\frac{\partial^2 T^i}{\partial \chi^2}\,, \quad \chi^{i,1} < \chi < \chi^{i,2}\,, \quad \tau > 0\,, \quad i \in \{\text{cov, fl}\}\,; \quad T^i = T^{sh}\,, \quad \chi^{i,1} < \chi < \chi^{i,2}\,, \quad \tau = 0\,, \quad i \in \{\text{cov, fl}\}\,;$$

$$\lambda^i\,\frac{\partial T^i}{\partial \chi} = q^{i,1}\,, \quad \chi = \chi^{i,1}\,, \quad \tau \ge 0\,, \quad i \in \{\text{cov, fl}\}\,; \quad \lambda^{cov}\,\frac{\partial T^{cov}}{\partial \chi} = \alpha_1^{cov,2}\,(T^{cov} - T^{sh})\,, \quad \chi = \chi^{cov,2}\,, \quad \tau \ge 0\,; \tag{7}$$

$$\lambda^{fl}\,\frac{\partial T^{fl}}{\partial \chi} = 0\,, \quad \chi = \chi^{fl,2}\,, \quad \tau \ge 0\,.$$

It must be noted here that the replacement of (7) by heat-transfer equations will introduce a significant error into the calculation of the heat loss by virtue of the above properties of the enclosing structures.

The quantities $q^{cov,1}$ and $q^{fl,1}$ are determined analogously to $q^{pr,1}$. The temperature of the interior surface of the chamber walls can be considered to be equal to the temperature of the upper surface of the floor [2].

The state variables of the steam-air medium of the chamber, i.e., T^{pl} (h^{pl}, M^{pl}_{st}, M^{pl}_{air}), P^{pl} (h^{pl}, M^{pl}_{st}, M^{pl}_{air}), and P^{pl}_{st} (h^{pl}, M^{pl}_{st}, M^{pl}_{air}), are determined from the equations of component-by-component material balances of the mixture and the equation of energy balance

$$\frac{dM^{pl}_{air}}{d\tau} = (G^{sh,1}_{mixt} + G^{sh,2}_{mixt})\,y^{sh}_{air} - (G^{pl,1}_{mixt} + G^{pl,2}_{mixt})\,y^{pl}_{air}\,,$$

$$\frac{dM^{pl}_{st}}{d\tau} = G^{main}_{st} + (G^{sh,1}_{mixt} + G^{sh,2}_{mixt})\,y^{sh}_{st} - (G^{pl,1}_{mixt} + G^{pl,2}_{mixt})\,y^{pl}_{st} + G^{pr,2}_{water} - G^{pr,1}_{st} - G^{cov,1}_{st} - G^{fl,1}_{st} - G^{f}_{st}\,,$$

$$\frac{dh^{pl}M^{pl}}{d\tau} = G^{main}_{st}h^{main} + (G^{sh,1}_{mixt} + G^{sh,2}_{mixt})\,h^{sh} - (G^{pl,1}_{mixt} + G^{pl,2}_{mixt})\,h^{pl} + (G^{pr,2}_{water} - G^{pr,2}_{st})\,h^{pr,2}_{sat} -$$

$$- G^{pr,1}_{st}h^{pr,1}_{sat} - G^{cov,1}_{st}h^{cov,1}_{sat} - G^{fl,1}_{st}h^{fl,1}_{sat} - G^{f}_{st}h^{pr,1}_{sat} - Q^{pr,2} - Q^{pr,1} - Q^{cov,1} - Q^{fl,1} - Q^{f} \tag{8}$$

with the following boundary conditions:

$$M^{pl}_{air} = M^{pl}_{air0}\,, \quad M^{pl}_{st} = M^{pl}_{st0}\,, \quad h^{pl}M^{pl} = h^{pl}_0 M^{pl}_0\,, \quad \tau = 0\,.$$

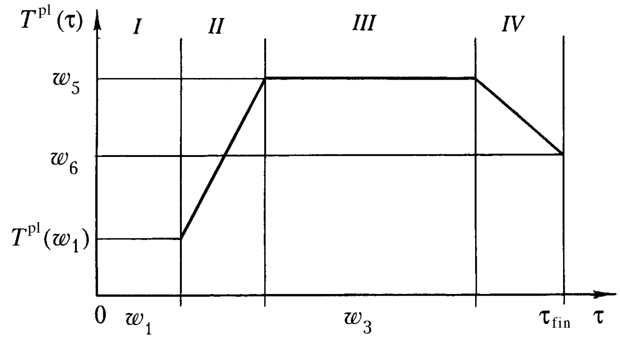

Fig. 2. General view of a typical temperature regime of heat and moist treatment of reinforced-concrete products: I) preliminary curing; II) heating; III) isothermal curing; IV) cooling.

The amount of water $G_{\text{water}}^{\text{pr},2}$ evaporating in a unit time from the open surface of the products and the flow rate of the vapor $G_{\text{vap}}^{\text{pr},2}$ condensing to the upper surface of the reinforced-concrete products is determined in the form

$$G_{\text{water}}^{\text{pr},2} = \begin{cases} S^{\text{pr},2} g_{\text{water}}^{\text{pr},2}, & P_{\text{st}}^{\text{pl}} < P_{\text{sat}}^{\text{pr},2}, \\ 0, & P_{\text{st}}^{\text{pl}} \geq P_{\text{sat}}^{\text{pr},2}, \end{cases} \qquad G_{\text{st}}^{\text{pr},2} = \begin{cases} S^{\text{pr},2} g_{\text{st}}^{\text{pr},2}, & P_{\text{st}}^{\text{pl}} > P_{\text{sat}}^{\text{pr},2}, \\ 0, & P_{\text{st}}^{\text{pl}} \leq P_{\text{sat}}^{\text{pr},2}. \end{cases} \tag{9}$$

Analogously we find the values of the flow rate $G_{\text{st}}^{\text{pr},1}$, $G_{\text{st}}^{\text{cov},1}$, and $G_{\text{st}}^{\text{fl},1}$:

$$G_{\text{st}}^{i,1} = \begin{cases} S^{i,1} g_{\text{st}}^{i,1}, & P_{\text{st}}^{\text{pl}} > P_{\text{sat}}^{i,1}, \\ 0, & P_{\text{st}}^{\text{pl}} \leq P_{\text{sat}}^{i,1}, \end{cases} \qquad i \in \{\text{pr, cov, fl}\}, \tag{10}$$

the area $S^{\text{fl},1}$ is made up of the area of the upper surface of the floor and the area of the interior surface of the chamber walls.

The quantities G_{st}^{f} and Q^{f} with account for the assumption $T^{\text{f}} = T^{\text{pr},1}$ are determined as

$$G_{\text{st}}^{\text{f}} = \begin{cases} \dfrac{Q^{\text{f}} g_{\text{st}}^{\text{pr},1}}{q_1^{\text{pr},1} + r_{\text{st}}^{\text{pr},1} g_{\text{st}}^{\text{pr},1}}, & P_{\text{st}}^{\text{pl}} > P_{\text{sat}}^{\text{pr},1}, \\ 0, & P_{\text{st}}^{\text{pl}} > P_{\text{sat}}^{\text{pr},1}, \end{cases} \qquad Q^{\text{f}} = c^{\text{f}} M^{\text{f}} \frac{dT^{\text{pr},1}}{d\tau}. \tag{11}$$

The heat fluxes resulting from the convective transfer of heat from the steam-air volume to the products and to the cover, walls, and floor of the chamber are found in the form

$$Q^{\text{pr},2} = \begin{cases} S^{\text{pr},2} q_1^{\text{pr},2}, & T^{\text{pl}} \geq T^{\text{pr},2}, \\ -S^{\text{pr},2} q_2^{\text{pr},2}, & T^{\text{pl}} < T^{\text{pl},2}, \end{cases} \qquad Q^{i,1} = \begin{cases} S^{i,1} q_1^{i,1}, & T^{\text{pl}} \geq T^{i,1}, \\ -S^{i,1} q_2^{i,1}, & T^{\text{pl}} < T^{i,1}, \end{cases} \qquad i \in \{\text{pr, cov, fl}\}. \tag{12}$$

The results of experimental determination of the coefficients of external heat and mass transfer in the steam-air medium of the chamber as functions of the temperature head, pressure in the chamber, composition of the steam-air medium, and orientation of the heat-exchange surface have been processed most completely in [4].

The obtained mathematical model of the process of heat and moist treatment of concrete and reinforced-concrete products is very complex and involves both ordinary differential equations and partial differential equations. Therefore, for the given mathematical model one employs the algorithm of its numerical solution involving the Runge–Kutta method of fourth order to calculate the system of ordinary differential equations and the marching method to solve the partial differential equations.

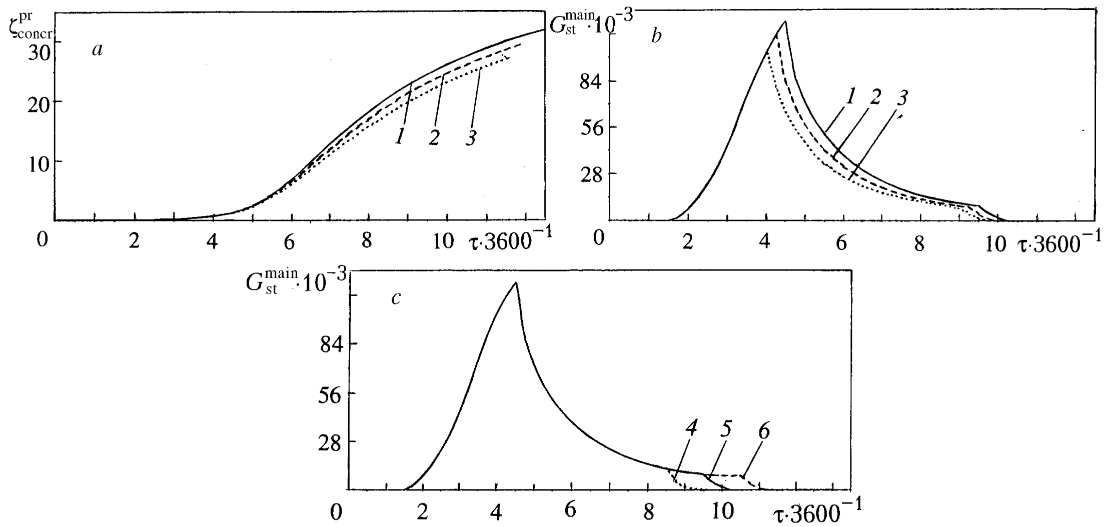

Fig. 3. Change in the ultimate strength of concrete (a) and in the flow rate of a heating steam (b and c) as a function of the temperature (curves 1–3) and the time of isothermal curing (curves 4–6): 1) $w_4 = 358$, 2) 353, and 3) 348 K; 4) $w_3 = 4{\cdot}3600$, 5) 5·3600, and 6) 6·3600 sec. ζ^{pr}_{concr}, MPa; G^{main}_{st}, kg/sec; τ, sec.

The mathematical model involves the unknown parameters and functions to be determined from experimental data by solution of the inverse problem. In solving this problem, we employed the approach of expansion of the unknown functions into truncated series according to the existing basis expressions and obtained the following results:

$$c_{concr} = 1.12\,,\quad \lambda_{concr} = 0.546 + 1.5{\cdot}10^{-4}T^{pr} + 0.011\nu\,,\quad \overline{\zeta^{pr}_{concr}} = 98.5\nu\,,\quad a_1 = 9.7\,,\quad a_2 = 1102\,.$$

A distinctive feature of simulation investigations of a periodic process of heat and moist treatment is that in the first step of typical technological regimes (time interval $[0, w_1]$ (Fig. 2)), the vector of the input and output fluxes is determined completely:

$$G^{main}_{st} = 0\,,\quad G^{sh,1}_{mixt} = 0\,,\quad \tau \in [0, w_1]\,;$$

$$G^{sh,2}_{mixt} = \overline{G^{sh,2}}\eta\,(P^{sh} - P^{pl})\,,\quad G^{pl,2}_{mixt} = \overline{G^{pl,2}}\eta\,(P^{pl} - P^{sh})\,,\quad G^{pl,1}_{mixt} = \overline{G^{pl,1}}\eta\,(G^{sh,1}_{mixt})\,,\quad \tau \in [0, \tau_{fin}]\,.$$

In this step, we solve the primal problem of determination of the vector of the output coordinates of the mathematical model. For the remaining steps (time interval $(w_1, \tau_{fin}J])$ we solve the inverse problem, i.e., the remaining output coordinates and the newly introduced generalized flow rate of the heat-transfer agent G_{gen} are determined simultaneously $(-\overline{G^{sh,1}} \le G_{gen} < \overline{G^{main}})$ from the prescribed function of change of the temperature in the chamber $T^{pl}(\tau, \mathbf{w}, T^{fl}(w_1))$, $\tau \in [w_1, \tau_{fin}]$. The quantities G^{main}_{st} and $G^{sh,1}_{mixt}$ for $\tau \in [w_1, \tau_{fin}]$ in accordance with the technological constraint $G^{main}_{st}G^{sh,1}_{mixt}$ are found in the form

$$G^{main}_{st} = G_{gen}\eta\,(G_{gen})\,,\quad G^{sh,1}_{mixt} = -\,G_{gen}\,(1 - \eta\,(G_{gen}))\,.$$

To solve the inverse problem based on (8) we obtained the equation describing a temperature change in the plant; this equation was solved relative to G_{gen}.

Figures 3 and 4 show the basic results of a simulation investigation of the process of heat and moist treatment of reinforced-concrete products, based on which we have drawn the following conclusions.

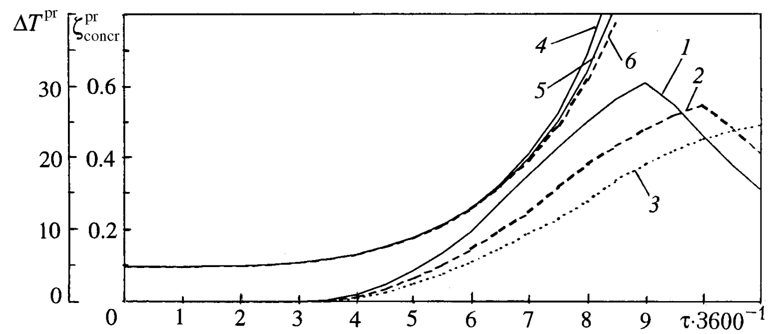

Fig. 4. Change in the temperature difference (curves 1–3) and in the ultimate strength of concrete (curves 4–6) as a function of the rate of increase of the temperature: 1 and 4) $w_4 = 0.005$ K/sec; 2 and 5) 0.006; 3 and 6) 0.07. ΔT^{pr}, K; ζ^{pr}_{concr}, MPa; τ, sec.

Of the technological parameters of the investigated regimes, the temperature of isothermal curing exerts the greatest influence on the ultimate strength of products (Fig. 3a). However the increase in the temperature has the most substantial effect on the increase in the steam flow rate (Fig. 3b) and in the expenditure of electric energy. The great influence exerted by the parameter w_5 on the expenditure of electric energy is explained by the fact that a temperature increase in the plant leads to an increase in the cooling portion and hence the operating time of the electric fan. It should also be taken into account that an increase in the temperature of isothermal curing can lend to the overheating of products (as a result of which low-strength compounds are formed) and to a decrease in the mechanical indices of the products.

The next technological parameter exerting a substantial influence on the ultimate strength of products is the time of isothermal curing w_3. A comparison of the plots in Fig. 3b and c shows that the change in the steam flow rate is insignificant (the change in the expenditure of electric energy here is even less significant), i.e., energy expenditure can be decreased further by increasing the time of isothermal curing and decreasing the temperature in the plant. However, an increase in the time of heat and moist treatment and, in particular, in the parameter w_3 can lead to a decrease in the turnover of forms and accordingly to an increase in the specific quantity of metal.

As the rate of increase of the temperature (parameter w_2) increases, the temperature gradient in the products sharply increases and, in particular, the temperature difference of the upper surface and the center of the products $\Delta T^{pr} = T^{pr}(\chi^{pr,2}) - T^{pr}(\chi^{pr,2} - (\chi^{pr,2} - \chi^{pr,1})/2)$ (Fig. 4); we observe an insignificant improvement in the strength of the products, which can lead to inadmissible stresses in the hardening concrete. As the rate of decrease of the temperature (parameter w_4) increases, we observe a certain retardation of the process of hardening of the concrete with simultaneous increase in the difference ΔT^{pr}, which can also lead to inadmissible stresses.

The influence of the parameters w_1 and w_6 on the kinetics of the processes occurring in the course of heat and moist treatment of reinforced-concrete products is the least significant. However, during the preliminary curing, the products are gaining a certain initial strength required for the stresses developing in the period of heating to be taken. The increase in the parameter w_6 can lead to significant irreversible strains in the period of demolding of reinforced-concrete products due to the inadmissible temperature difference of the open surface of the products and the air in the production-floor area.

Thus, the simulation investigations carried out have shown good agreement with experimental investigations and the possibility of using the obtained mathematical model in problems of optimal control of the process of heat and moist treatment of reinforced-concrete products.

NOTATION

M, mass, kg; P, pressure, Pa; T, temperature, K; G, mass flow rate, kg/sec; Q, heat flux, J/sec; S, area, m^2; h, specific mass enthalpy, J/kg; q, heat-flux density, J/(m^2·sec); g, specific mass flow rate, kg/(m^2·sec); r, heat of vaporization, J/kg; m, volume concentration, kg/m^3; y, mass concentration, kg/kg; c, specific mass heat capacity, J/(kg·K); a_1, preexponential factor, sec^{-1}; a_2, subexponential factor, K; b, proportionality coefficient, J/kg;

$\mathbf{w} = \{w_1, ..., w_6\}$; w_1 and w_3, time of preliminary and isothermal curing, sec; w_2 and w_4, velocity of the heating and cooling portions, K/sec; w_5 and w_6, temperature of isothermal curing and of the end of the cooling portion, K; α, heat-transfer coefficient, J/(m^2·sec·K); β, mass-transfer coefficient, kg/(m^2·sec·Pa); ρ, density, kg/m^3; λ, thermal-conductivity coefficient, J/(m·sec·K); ζ, ultimate uniaxial-compression strength, MPa; ν, degree of hydration of concrete; τ, time, sec; χ, space coordinate, m; η, Heaviside function. Subscripts: st, unsaturated steam; sat, saturated steam; air, air; water, water; mixt, steam-air mixture; concr, concrete; cem, cement; gen, "generalized" heat-transfer agent; 0, initial instant of time; fin, final instant of time. Superscripts: pl, steam-air volume of the plant; pr, product; f, form; cov, cover; fl, floor; sh, production floor-area (shop); main, main steam line; 1 and 2, numbers of fluxes or heat-exchange surfaces; i, element of the set of superscripts; $\overline{\quad}$, highest limiting value of the variable.

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